# MANONMANIAM SUNDARANAR UNIVERSITY, TIRUNELVELI DEPARTMENT OF MATHEMATICS

## **Integrated M.Sc Mathematics (CBCS)**

(For those who join the programme from the Academic year 2019-20)

## **REGULATIONS, SCHEME OF EXAMINATIONS AND SYLLABUS**

#### A. Regulations

#### A1: Programme objectives:

- ➤ The idea of the program is to attract the young talents to Mathematics keeping in line with the policy of the Government of India to promote education in pure sciences. The syllabus is framed keeping this goal in mind.
- ➤ Elective course in the fourth and fifth years are planned to suit competitive examinations like NET and SLET.
- > Students undergoing this programme will have the opportunity of choosing research / teaching at leading research institutions or a career in corporate sectors.
- ➤ To enable the students to have a thorough exposure to the different branches of Mathematics so as to gain a comprehensive knowledge of Mathematics.
- ➤ To cultivate logical thinking and analytical skills which sharpens their concentration and provides patience to grapple with life outside the campus.

#### **A2: Duration of the Course:**

The duration of the programme is 5 years under choice based credit system. The minimum number of credits to be earned during the first three years of the prorammme is 140. Students who have passed all the papers for the first three years of the programme will be given a B.Sc Degree at the end of the third year. Students who have passed all the courses for the fourth and fifth years of the programme will be given M.Sc Degree. Students have an option to exit from the program with a B.Sc Degree alone.

Students who have passed all the courses for the first three years of the programme alone will be permitted to continue the programme in the 4<sup>th</sup> year. Students who are not successful at the end of the third year will not be permitted to continue or rejoin the programme in future under the integrated mode. The minimum number of credits to be earned during the last two years of the programme to get M.Sc Degree is 90.

#### A3: Eligibility norms for admission:

Those who seek admission to Integrated M.Sc. Mathematics Degree programme

must have passed the Higher Secondary Examinations (+ 2) conducted by the Board of Higher Secondary Examination, Tamil Nadu with the subjects Mathematics, Physics and Chemistry or an equivalent programme of study with 60% of marks in Mathematics (55% for SC/ST/SCA applicants)

**A4: Mode of Admission:** An entrance examination (objective type questions) will be conducted for eligible applicants. The merit list will be prepared for a total of 100 marks with 50 marks for entrance examination and 50 marks for mathematics paper in + 2. Then admission will be based on merit and reservation policy of the Government of Tamil Nadu.

#### B. Scheme of examinations:

Each course is for 100 marks with Internal 25 marks and External 75 marks.

For Semesters I - VI, the internal assessment comprises of 2 components - 20 marks for written test ( average of the best two of 3 tests ) and 5 marks for Assignment.

For Semesters VII - X, the internal assessment comprises of 3 components -15 marks for written test (average of the best two of 3 tests), 5 marks for Seminar and 5 marks for Assignment.

The Question paper pattern for end semester examination is as follows:

Section A -  $10 \times 1 = 10$  (Objective type questions)

Section B -  $5 \times 5 = 25$  (Internal choice questions)

Section C -  $5 \times 8 = 40$  (Internal choice questions)

The duration of the examination is 3 hours. Passing minimum in the external examination is 50% (that is 38 out of 75). Passing minimum in the aggregate (internal and external marks put together) 50% (that is 50 out of 100). No passing minimum for the internal examinations.

Examination, evaluation and classification will be made as per the rules and regulations of the University in force.

#### C. SYLLABUS

The syllabi for Part – I (Tamil ) and Part – II (English) courses are as framed by the Department of Tamil and English respectively. The syllabi for Part IV (Environmental Studies and Value based education etc) shall be as framed by the respective Boards constituted for this purpose.

The structure and syllabi of other subjects are as below:

	Part	Su	Sub.	Subject Title	Hrs/	L	Т	Р	Cre
Sem	1/11/111	b.	Status	(5)	week	Hrs/	Hrs/	Hrs/	dits
(1)	/IV/V	No	(4)		(6)	week	week	week	(10)
	(2)	(3)				(7)	(8)	(9)	
I	I	1	Language	Tamil / Other Language	4	4	0	0	4
	II	2	Language	English	4	4	0	0	4
	Ш	3	Core -1	Theory of Equations	4	4	0	0	4
	Ш	4	Core- 2	Differential Calculus	4	4	0	0	4
	Ш	5	Allied – I	Physics – I	3	3	0	0	3
	Ш	6	Allied - I	Practical	4	0	0	4	2
	IV	7	Common	Environmental Studies	2	2	0	0	2
II	I	8	Language	Tamil / Other Language	4	4	0	0	4
	II	9	Language	English	4	4	0	0	4
	Ш	10	Core -3	Analytical Geometry	4	4	0	0	4
	Ш	11	Core- 4	Vector Calculus	4	4	0	0	4
	Ш	12	Allied – I	Physics – II	3	3	0	0	3
	III	13	Allied – I	Physics – II ( Practical)	4	0	0	4	2
	IV	14	Common	Value based education /	2	2	0	0	2
				Social Harmony					
	I	15	Language	Tamil/ Other Language	4	4	0	0	4
	II	16	Language	English	4	4	0	0	4
	Ш	17	Core -5	Real Analysis	4	4	0	0	4
	Ш	18	Allied – II	Chemistry – I	3	3	0	0	3
III	Ш	19	Allied – II	Practical	4	0	0	4	2
	Ш	20	Skill		4	4	0	0	4
			based	Programming in C					
			Subject						
	IV	21	Non-major	Basic Mathematics	3	3	0	0	3
			Elective						
	IV		Mandatory	Yoga	2	2	0	0	2
	I	22	Language	Tamil / Other Language	4	4	0	0	4
	II	23	Language	English	4	4	0	0	4

IV	Ш	24	Core - 6	Differential Equations	4	4	0	0	4
	Ш	25	Allied – II	Chemistry – II	3	3	0	0	3
	III	26	Allied – II	Chemistry II – Practical	4	0	0	4	2
	Ш	27	Skill		4	4	0	0	4
			based	Latex and Matlab					
			Subject						
	IV	28	Non-major	Discrete Mathematics	3	3	0	0	3
			Elective						
	IV		Mandatory	Computers for digital era	2	2	0	*	2
	V		Extension	NCC/NSS/YRC/YWF	2	0	0	2	1
V			activity						
	III	29	Core – 7	Abstract Algebra	4	4	0	0	4
	III	30	Core – 8	Metric Spaces	4	4	0	0	4
	Ш	31	Core – 9	Statics	4	4	0	0	4
	Ш	32	Major		3	3	0	0	3
			Elective-I						
	Ш	33	Major		3	3	0	0	3
			Elective-II						
	IV	34	Skill	Personality Development/	2	2	0	0	2
			based	Effective Communication/					
			Subject	Youth Leadership					
			(common)						
VI	III	35	Core -10	Complex Analysis	4	4	0	0	4
	III	36	Core -11	Linear spaces	4	4	0	0	4
	III	37	Core -12	Graph Theory	4	4	0	0	4
	III	38	Core -13	Dynamics	4	4	0	0	4
	III	39	Major		3	3	0	0	3
			Elective-III						
		40.	Group		6	4	2	0	6
			Project						
	·	1	1	Total	156				140

<sup>\* 10</sup> hours for practical.

L- Lecture T- Tutorials P- Practical

## **List of Major Elective Papers**

- 1. Elementary Number Theory
- 2. Statistics I
- 3. Statistics II
- 4. Numerical Methods
- 5. Linear Programming
- 6. Combinatorial Mathematics
- 7. Coding Theory
- 8. Operations Research
- 9. Mathematical Programming Using JAVA
- 10. Integral transforms and applications
- 11. Special Functions
- 12. Mathematical Logic
- 13. Astronomy
- 14. Mathematical Modeling

#### **DETAILED SYLLABUS**

#### Semester I

## **Core Paper - 1: Theory of Equations (60 hours)**

**Preamble:** Students at the higher secondary level know the quadratic formula to solve polynomial equation of degree two. This course is aimed to introduce various methods, geometrically and analytically, to solve polynomial equations of degree greater than two. The out come of the course is a thorough understanding of the relation between the roots and the coefficients of the equation.

**Unit I:** Theory of equations: Remainder theorem - imaginary roots - irrational roots - relations roots and coefficients (12 hours)

**Unit II:** Symmetric function of the roots - Sum of the powers of the roots - Newton's theorem **(12 hours)** 

**Unit III:** Transformations of equations - Transformations of equations in general (12 hours)

**Unit IV:** Reciprocal equations - Descarte's rule of signs - Rolle's theorem - multiple roots. **(12 hours)** 

**Unit V:** The Cubic equation - Cardan's method - The Biquadratic equation - Ferrari's method. **(12 hours)** 

**Text Books:** 1. T.K.Manicavachagam Pillay, T. Natarajan and K.S. Ganapathy, S.Viswanathan - *Algebra (Volume I)* - (Printers and Publishers) Pvt. Ltd. 2008

(Unit I: Chapter 6 - Sections 1 to 12, Unit II: Chapter 6 - Sections 13 to 15, 21 Unit III: Chapter 6 - Sections 16, 24, 25, 26)

2. S. Arumugam and A. Thangapandi Isaac, Set theory, Number System and theory of equations, New gamma publishing house, 1992

(For Unit IV; Chapter 5 - Sections 5.8 and 5.9)

## **Core Paper - 2: Differential Calculus (60 hours)**

**Preamble:** This course will enable the student to learn the different concepts of differential and integral calculus. Specific outcome of learning: The learner will gain knowledge of differentiation and its applications; The learner will acquire basic knowledge of integration; and the learner will know the concepts of change of variables.

**Unit I:** Limit of a real function - Continuity.

(12 hours)

**Unit II:** Derivative - Simple examples - Rolle's theorem - Mean value theorem. (12 hours)

**Unit III:** Envelopes - Curvature - Circle, radius and centre of curvature - Radius of Curvature in Cartesian, parametric and polar co-ordinates. (12 hours)

**Unit IV:** Evolutes and involutes - Pedal equation of a curve.

(12 hours)

**Unit V:** Linear asymptotes - Tracing of curves.

(12 hours)

#### **Text Books:**

- 1. Richard R. Goldberg, *Methods of Real Analysis*: Oxford & IBH Publishing, 1963(for Units I and II)
- 2. S. Narayanan and T.K.Manicavachagom Pillay, *Calculus Vol. I*, S.Viswanathan Printers and Publishers Pvt. Ltd., 2010 (for Units III, IV and V; Chapters X, XI, XIII).

Allied – I
Physics - I
Physics I: Practical

(Syllabus as prepared by Physics BoS).

#### Semester II

## **Core Paper - 3: Analytical Geometry (60 hours)**

**Preamble:** This course covers the topics equation of planes in standard form, normal form and intercept form, image of a line, shortest distance between skew lines, coplanar lines, sphere, cone and cylinder. This course does not require any prerequisite except the basic ideas on the two dimensional geometry. After completing this course, the students will be able to understand the three dimensional coordinate system and relative positions of celestial bodies in our galaxy.

Unit I: . Direction cosines - direction ratios - equation of planes - standard form - normal form - intercept form(12 hours)

**Unit II:** Angle between two planes - Lines - symmetrical form. Angle between two planes - image of a point - image of a line (12 hours)

Unit III: Skew lines - shortest distance between two lines - coplanar lines. (12 hours)

**Unit IV:** Sphere - plane section of a sphere - tangent plane - intersection of two spheres - intersection of a plane with sphere (12 hours)

Unit V: Cone and Cylinder. (12 hours)

#### Text books:

 T.K Manicavachagom Pillay and T. Natarajan, A text book of Analytical Geometry - Part II - Three dimensions, S.Viswanathan Printers and Publishers Pvt. Ltd. - 2008.
 (for Units III, IV and V, Chapters I to IV and Chapter V- Sections 1 to 7.)

#### Core Paper - 4: Vector Calculus (60 hours)

**Preamble:** Most of the problems in Physics and Engineering involves the notion of flow and vector field like magnetic field, fluid flow and electric flow etc. The aim of this course is to study the notion of differentiation and integration of a vector field, particularly in three dimension. It motivates the students to approach the problems geometrically. The out put is, students will have a good knowledge in Calculus of real variable.

Unit I: Vector differentiation - gradient - divergence - curl - solenoidal and irrotational vector -

formulae involving gradient, divergence and curl. (12 hours)

**Unit II:** Vector integration - line integral - double and triple integral. (12 hours)

Unit III: Jacobians - change of variables . (12 hours)

Unit IV: Line and surface integrals (12 hours)

Unit V: Theorems of Green, Stokes and Gauss. (12 hours)

**Text book:** S. Arumugam and A.Thangapandi Isaac, *Vector calculus*, New gamma publishing house, 2006 (For units I, II and III- Chapters 5, 6 and & 7).

## Allied – I Physics II Physics II - Practical

(Syllabus as prepared by Physics BoS).

## Allied - I

## Mathematics I (45 hours)

(For students who studied Mathematics at + 2)

**Preamble**: This course aims to provide basic concepts in many areas of pure and allied mathematics such as Algebra, Differential Equations and Theory of equations. It facilitates the students to undertake further studies in mathematics and also helps to face competitive examinations.

**Unit I**: Theory of equations - Relation between roots and coefficients - symmetric function of the roots in terms of coefficients. (9 hours)

**Unit II**: Matrices - Characteristic equation of a matrix - Eigen values and eigen vectors - Cayley Hamilton theorem and simple problems. **(10 hours)** 

**Unit III**: Differential equation of first order but of higher degree - Equations solvable for p, x, y. **(10 hours)** 

Unit IV: Laplace transformation - Inverse Laplace transform (8 hours)

**Unit V:** Solving linear differential equations using Laplace transforms. (8 hours)

Text: S. Arumugam and Others, Allied Mathematics.

## Mathematics I: Practical (30 hours) MATLAB

**Preamble:** MATLAB stands for MATrix LABoratory. The objective of the course is to learn the basics of the programming language MATLAB such as tokens, Numerical computation and visualization. After completing this course, the student will be able to provide an interactive environment with hundreds of built-in functions for technical computation, graphics and animation. Best of all, also provides easy extensibility with its own high level programming language. The name

- 1. To find the roots of the quadratic polynomial.
- 2. To find the reciprocal roots of the polynomial.
- 3. To find the roots of the symmetric function.
- 4. To find the eigen value of the square matrix.
- 5. To find the eigen vector of the square matrix.
- 6. Solve the first order differential equation using MATLAB.
- 7. Solve the second order differential equation using MATLAB.
- 8. To find integration when limits are not given.
- 9. To find integration when limits are given.
- 10. Simple 2-D, 3-D plots by using MATLAB.

## Mathematics II (45 hours)

(For students who studied Mathematics at + 2)

**Preamble:** This course provides all the fundamentals in Vector Calculus which is needed wherever approximation plays a role. The pre-requisite for this course is a basic knowledge in mathematics at the Higher Secondary level. The output of this course is that students are enabled to have an entry into the portals of Analysis in almost all branches of mathematics.

**Unit I**: Vector differentiation - Gradient - Divergence and curl. (10 hours)

Unit II: . Evaluation of double and triple integrals. (10 hours)

Unit III: Vector integration - Line integral, (8 hours)

Unit IV: Surface and volume integrals. (8 hours)

**Unit 5**: Green's, Stoke's and Divergence Theorems (statement only) - simple problems.

(9 hours)

Text Books: 1. S. Arumugam & Others, Vector Calculus, New Gamma Publishing House, 2006

2. T.K.Manicavachagom Pillay, *Calculus (Volume II)*, S. Viswanathan (Printers and Publishers) 2010.

Mathematics – II Practical (30 hours)

#### **MATLAB**

**Preamble:** To learn the basics of the programming language MATLAB such as tokens, Numerical computation and visualization. After completing this course, the student will be able to provides an interactive environment with hundreds of built-in functions for technical computation, graphics and animation. Best of all, also provides easy extensibility with its own high level programming language. The name MATLAB stands for MATrix LABoratory.

- 1. To calculate exponential and logarithm.
- 2. To calculate values of Trigonometric function.
- 3. Creating and working with arrays of numbers.
- 4. To use trigonometric functions with array arguments.
- 5. To find sum of geometric series.
- 6. To compute dot product of vectors and cross product of vectors.
- 7. To compute box and vector triple product.
- 8. To compute angle between two vectors.
- 9. To calculate interest of your money.
- 10. To define the function using variable as symbol.
- 11. To find out left and right limit of the given function.
- 12. To find limit of a function.

#### Semester III

Core Paper- 5: Real Analysis (60 hours)

**Pre-amble:** This course provides all the fundamentals in convergence of sequences and series which is a necessity in pure and applied mathematics, particularly wherever approximation plays a role. The prerequisite is basic knowledge in mathematics provided at the Higher Secondary level. This course provides the student an entry into the portals of Analysis in almost all branches of mathematics.

**Unit I:** Bounded sets - upper and lower bounds - LUB axiom - Archimedean property - Density of rational and irrationals - Positive n<sup>th</sup> root of a positive real number. **(12 hours)** 

Unit II: Sequences - Bounded sequences - monotonic sequences - convergent sequences - divergent and oscillating sequences (12 hours)

Unit III: The algebra of limits. Behaviour of monotonic sequences. (12 hours)

**Unit IV:** Some theorems on limit - sub sequences - limit points - Cauchy sequences - Cauchy general principle of convergence of sequence (12 hours)

Unit V: Series - infinite series - comparison test - D'Alembert's ratio test - Root test - Cauchy's condensation test.(12 hours)

**Text book:** S.Arumugam and A. Thangapandi Isaac, *Sequences and series*, New gamma publishing house, 2010.

#### Allied II

## Chemistry I

Chemistry I - Practical

(Syllabus as prepared by the BoS in Chemistry)

## **Skill Based Elective**

#### PROGRAMMING IN C (60 hours)

**Preamble:** The objective of this course is to provide the basics of the computer programming terminologies. This course does not require any prerequisite. After completing this course, the student will get problem solving abilities using computers and they will write simple 'C' programs using decision making statements, loop statements, pointers and user defined functions.

**Unit I:** Constants - variables - Data types - operations and Expressions - managing input and output operations (15 hours)

**Unit II:** Decision making and branching - Decision making and looping (15 hours)

**Unit III:** Arrarys - Handling of character strings (10 hours)

Unit IV: Structures and unions (10 hours)

Unit V: Pointers. (10 hours)

**Text Book:** E. Balagurusamy, *Programming in ANSI C,* IV Edition, TMH Publishing Company, 2008, Limited Chapters 2 to 7, 9 - 11.

## Non - major Elective

## **Basic Mathematics (45 hours)**

**Preamble:** This course introduces elementary concepts of mathematics for other major students. It covers the topic which include numbers, H.C.F. and L.C.M. of numbers, Decimal fraction, BODMAS Rule, square root, cube root, perfect square and perfect cube. This course does not require any prerequisite. Upon successful completion of this course, the students will be able to get quick solution for problems arising in their non mathematics subjects.

**UNIT I:** Numbers - Face value and place value of a digit in a number - test of divisibility, Applications of algebraic formulae, unit digit - series. (10 hours)

UNIT II: H.C.F. and L.C.M. of numbers - factorization method - common division method, H.C.F.and L.C.M. of decimal fraction - comparison of fractions. (10 hours)

**UNIT III:** Decimal fraction- conversion of decimal into vulgar fraction-operation son decimal fractions- comparison of fractions-recurring decimal-mixed recurring decimal. **(9 hours)** 

UNIT IV: Simplification - BODMAS Rule - modulus of a real number - virnaculum - Some real life problems - missing numbers in the expression.(8 hours)

UNIT V: Square root and cube root - finding square root by factorization method- perfect square and perfect cube.(8 hours)

Text Book: R.S. Aggarwal, Quantitative Aptitude - (2014), S. Chand & Co., Chapters 1 to 5

#### Semester IV

#### **Core Paper 6 - Differential Equations (60 hours)**

**Preamble:** Most of the physical problems, when rate of change of certain things occurs, can be reduced to an equation called differential equation. It is of our practical interest to solve this type of equations to understand the physical phenomena of the nature. This course provides a basic understanding of Differential Equations and techniques to solve them. Prerequisite: Calculus

Unit I: Equation of First Order and First Degree - Variable separable method -

Homogeneous equations - non-homogeneous equations - Exact differential

equations - Linear equations (12 hours)

**Unit II:** Linear Equations of Higher Order with Constant Co-efficients - Method of finding

Complementary Functions - Particular Integrals (12 hours)

**Unit III:** Linear Equations with variable co-efficients - The general solution in terms of

known integrals of C.F. - Removal of first derivative - The method of variation of

parameter - Simultaneous linear differential equations (12 hours)

Unit IV: Formation of PDE - Some standard forms of first order PDE (12 hours)Unit V: Solving first order PDE - Lagrange's method - Charpit's method (12 hours)

#### **Text Books**

1. S. Arumugam and Isaac, *Differential Equations and Applications*, New Gamma Publishing House, 1999.

(Unit I: 1.1 - 1.5, Unit II: 2.1 - 2.3, Unit III: 2.4 - 2.5, Unit IV, V: 4.1 - 4.5)

#### Allied II

#### Chemistry II

Chemistry II - Practical

(Syllabus as framed by the BoS in Chemistry)

## **Skill Based Elective**

#### LaTex and MATLAB (60 hours)

**Preamble:** This course aims to impart the programming concepts of matlab and latex. Specific outcome of learning: The learner will be

- > Able to use Matlab for interactive computations.
- > Able to draw 2D and 3D graphs.
- Able to applying programming techniques to solve problems at advanced level.
- > Understand richness of Latex rather than using M.S word for documentation.
- Proficient in documentation using mathematical symbols, graphs and tables.

**Unit I:** Document Layout and organization - Document class, Page style, Parts of the document, Text formatting, TeX and its offspring, what's different in latex 2 (Distinguishing LaTeX 2 and basics of LaTeX file). (12 hours)

**Unit II:** Commands and environments - command names and arguments, Environments, Declarations, Lengths, Special characters, Fragile Commands, Exercise. Table of contents, Fine - Tuning text, Word division, Labelling, Referencing, Displayed text - Changing font, Centering and identifying, Lists, Generalized Lists, Theorem like declarations, Tabular stops, Boxes.

(14 hours)

Unit III: Tables, Printing literal text, Footnotes and marginal notes. Drawing pictures using LaTeX, Mathematical formulas - mathematical environments, Main elements of math mode, Mathematical symbols, Addition elements, Fine - tuning Mathematics. (12 hours)
 Unit IV: Introduction - Basics of MATLAB, Input-output, File types - Platform dependence - General commands. (10 hours)

**Unit V:** Interactive Computation: Matrices and Vectors - Matrix and array operation-creating and using inline functions - using built - in functions and on-line help - saving and loading data - plotting simple graphs, Basic programming in MATLAB, creating cps files using MATLAB.

(12 hours)

#### **Text Books:**

- 1. H. Kopka and P.W. Daly, *A Guideline to LaTeX* -, Third edition, Addison Wesley, London, 1999.
- 2. Rudra Pratap, Getting started with MATLAB A quick Introduction for Scientists and Engineers, Oxford University Press, 2003

## **Non- Major Elective**

#### **Discrete Mathematics**

(45 hours)

**Preamble:** This course introduces the concepts of mathematical logic, set theory, relations and functions. This course requires the basic knowledge of set theory and functions at higher secondary school level. On successful completion of the course, students will be able to apply the above concepts in their further study in computer science.

**Unit I:** Mathematical logic: Logical statement or proposition- type of propositions- the propositional calculus - the negation of a proposition- disjunction- conjunction- tautologies and contradictions- logical equivalence - the algebra of propositions- conditional propositions - converse inverse and contrapositive propositions - the negation of a conditional proposition - biconditional propositions - arguments. **(10 hours)** 

**Unit II:** Set theory: Set- set designation- null sets and unit sets- special sets of numbers-universal set- subsets, proper subsets and equal sets- set operations- union operations-properties of union operation- intersection- properties of intersection operation. **(10 hours)** 

**Unit III:** Distributive properties - complementation- relative complement - properties of complement - properties of difference - symmetric difference- power set- Cartesian products.

(9 hours)

**Unit IV:** Relation and functions: Relation- equivalence relation- partition- partial order relation.

(8 hours)

**Unit V:** Function - inverse mapping- composition mappings - binary operations- countable and uncountable sets. **(8 hours)** 

#### Text book:

B.S. Vatssa, *Discrete Mathematics* - 3<sup>rd</sup> Edition , Wishwa Prakashan, 2009, Chapters 1, 2 (except 2.20) and 3.

#### SEMESTER V

#### Core Paper- 7: Abstract Algebra (60 hours)

**Preamble:** The objective of this course is to introduce the fundamental structures that are the needed for recent developments in Discrete Mathematics, Mathematical Physics, Computer Science etc. A basic knowledge of concepts like sets, relations, functions, mappings etc., are the prerequisite. The outcome of this course is to get a good knowledge of abstract ideas which is mandatory for higher level of study in Mathematics.

**Unit I:** Semigroups and groups - homomorphisms - subgroups and cosets (12 hours)

**Unit II:** Cyclic groups - Normal subgroups - quotient groups - isomorphism theorems - automorphisms. **(12 hours)** 

**Unit III:** Permutation groups: Cyclic decomposition - alternating group A<sub>n</sub> (12 hours)

**Unit IV:** Rings: Types of rings - Subrings and characteristic of a ring - additional examples of rings (12 hours)

Unit V: Ideals - homomorphisms - sum and direct sum of ideals - maximal and prime ideals.

(12 hours)

**Text book:** P. B. Bhattacharya, S.K.Jain and S.R.Nagpaul, *Basic Abstract Algebra*, Second Edition (1995), Cambridge University Press.

Unit I - Chapter 4 (Sections 1 - 3), Unit II - Chapter 4 (Section 4, Chapter 5 (Sections 1 - 3), Unit III - Chapter 7 (Sections 1 & 2), Unit IV- Chapter 9 (Sections 3 - 5), Unit V - Chapter 10 (Sections 1 - 4).

## Core paper - 8: Metric spaces (60 hours)

**Preamble:** This course is a pre-requisite for all advanced course in mathematics. It covers all the fundamentals of metric topology. This pre-requisite is a basic course in Real analysis which is usually covered in an undergraduate mathematics programme in the first or second semester. This course facilitates the student an entry into the portals of higher learning in mathematics.

**Unit I:** Metric spaces: Definitions and examples- Bounded sets in a metric space- open ball in a metric space- open sets- subspaces. **(12 hours)** 

**Unit II:** Interior of a set - Closed sets - closure, limit point, dense sets - complete metric space: Introduction - Completeness - Baire's category theorem. (12 hours)

Unit III: Continuity: Introduction - Continuity - homeomorphism. (12 hours)

**Unit IV:** Connectedness: Introduction- definition and examples - connected subsets of R-connectedness and continuity. (12 hours)

**Unit V:** Compactness: Introduction - compact space- compact subsets of R - equivalent characterization for compactness. (12 hours)

**Text Book:** S. Arumugam and Thangapandi Isaac, *Modern Analysis*(2010) - New gamma publishing house. Chapters 2, 3, 4 (except section 4.4),5 and 6.

## Core Paper 9 – Statics (60 hours)

**Preamble:** The object of this course is to introduce the basic laws, principles and postulates governing static system. On the successful completion of the course, students will be able to understand the basic concepts and principles of statics, analyze the mechanism of physical problems, and introduce the postulates governing static system.

Unit I: Forces acting at a point	(12 hours)
Unit II: Parallel forces and moments	(12 hours)
Unit III: Couples	(12 hours)
Unit IV: Equilibrium of three forces acting on a rigid body	(12 hours)
Unit V: Coplanar forces	(12 hours)

#### **Text Book:**

M.K. Venkataraman, A text book of Statics (2002), Agasthiar publications (Chapters 2-6).

#### Semester VI

## **Core Paper - 10: Complex Analysis (60 hours)**

**Preamble:** Complex Analysis is a subject which has interlink with most branches of Mathematics, such as Functional Analysis, Differential Equations, Algebra, Fluid Dynamics etc. Complex Analysis provides an interesting technique to solve operator equations. This course is aimed to provide sufficient knowledge on the analytic functions and the complex integrations. The series representation of complex analytic functions also included.

**Unit I**: nth roots of a complex number - circles and straight lines - regions in the complex plane - Riemann's stereographic projection. (12 hours)

**Unit II**: Differentiability - Cauchy Riemann equations - Analytic functions - Harmonic functions - Power series. (12 hours)

**Unit III**: Bilinear transformations - cross ratio - fixed points of Bilinear transformations - Mapping properties. (12 hours)

**Unit IV**:: Complex Integration - Definite integral - Cauchy's theorem - Cauchy's integral formula - higher derivative. (12 hours)

**Unit V**:: Residues, Cauchy's residue theorem - evaluation of definite integrals - Type I and Type II integrals. (12 hours)

Text Book: S. Arumugam and Isaac, Complex Analysis, Scitech Publications, 2011.

## Core Paper – 11: Linear spaces (60 hours)

Preamble: Linear spaces is a branch of mathematics that studies systems of linear equations and the properties of matrices. The concepts of linear algebra are extremely useful in physics, economics, social sciences, natural sciences, and engineering.

Unit 1: Vector Spaces: Definitions and Examples - Subspaces - Linear Transformation

(12 hours).

**Unit 2:** Span of a set-Linear Independence-Basis and dimension

(12 hours).

Unit 3: Rank and Nullity - Matrix of a Linear transformation-Inner product spaces: Introduction -Definition and Examples.

Unit 4: Orthogonality- Orthogonal Complement-Theory of Matrices: Simultaneous linear equations. (12 hours).

**Unit 5:** Characteristic equation and Cayley Hamilton Theorem

(12 hours).

Text Book: S. Arumugam and A.T.Isaac, Modern Algebra (2003), SCITECH Publications (India) PVT LTD, Chennai. **Sections:** 5.1-5.8, 6.0 to 6.3, 7.6 to 7.8.

#### Core Paper – 12: Graph Theory (60 hours)

Preamble: This course is a new branch of Mathematics which got its due recognition because of its diverse applications in computer science, chemistry, sociology. It is a part of Discrete Mathematics which deals with a finite set of objects. Pre-requsite is a reasonably good knowledge of the concepts like sets, relations, functions etc at the higher secondary leveli. Upon successful completion of this course, the students will able to use graph theory as a modelling to solve real life problems.

Unit I: Definition and Examples of Graphs - Degrees - subgraphs - isomorphism - independent sets and coverings - intersection graphs and line graphs - matrices - operation on graphs. (12 hours)

Unit II: Degree sequences - Graphic sequences - Walks - Trials and Paths - connectedness and components - connectivity. (12 hours)

Unit III: Eulerian graphs - Hamiltonian graphs - characterization of trees - centre of a tree -Matchings. (12 hours)

Unit IV: Definition and Properties of planar graphs - characterization of planar graphs chromatic number and chromatic index. (12 hours)

Unit V: Chromatic polynomials - definition and basic properties of digraphs - paths and connectedness in digraphs - digraphs and matrices. (12 hours)

Text Book: S. Arumugam and S. Ramachandran, Invitation to Graph Theory, (2001), Scitech Publications Pvt. Ltd. (Chapters 2 to 10).

## Core paper - 13: Dynamics (60 hours)

**Preamble:** The objective of this course is to introduce the basic laws, principles and postulates governing dynamic system. It covers the topics projectiles, collision of elastic bodies oblique impact of two smooth spheres, simple harmonic motion. On the successful completion of the course, students will be able to understand the basic concepts and principles of dynamics, analyze the mechanism of physical problems, and introduce the postulates governing dynamic system.

Unit I: Projectiles

Unit II: Velocity of the projectile

Unit III: Collision of elastic bodies

Unit IV: Oblique impact of two smooth spheres

Unit V: Simple Harmonic motion

(12 hours)

(12 hours)

**Text Book:** M.K. Venkataraman, *A text book of Dynamics*,(1990) Agasthiar publications (Chapters 6 (except 6.18), 8, and 10).

#### **MAJOR ELECTIVE PAPERS**

## 1. Elementary Number Theory (45 hours)

**Preamble:** This course is proposed for the compelling reason that number theory has become essential for Cryptology. Topics include the integers, divisibility, prime numbers, primality testing, factorization methods, congruences, arithmetical functions, Fermat's little theorem, Fermats's last theorem, arithmetical functions and so forth.

**Unit 1:** Divisibility Theory in the Integers: Early number theory-the division algorithm-the greatest common divisor-the Euclidean algorithm. **(9 hours)** 

**Unit 2:** The Diophantine equation - Primes and their distribution: The fundamental theorem of Arithematics-the Sieve of Eraosthenes - the Goldbach conjecture. **(9 hours)** 

**Unit 3:** The theory of congruences: basis properties of congruences - binary and decimal representations of integers - linear congruences and the Chinese remainder theorem.

(9 hours)

**Unit 4:** Fermat's theorem: Fermat's Little theorem and Pesudoprimes - Wilson theorem - The Fermat-Kraitchik factorization method. **(9 hours)** 

**Unit 5:** Number theoretic functions: the sum and number of divisors - the Mobius inversion function - the greatest integer function. **(9 hours)** 

**Textbook:** David M. Burton, Elementary number theory, Seventh Eds, Tata McGarw-Hill, New Delhi (2012).

Sections: 2.1 to 2.5, 3.1 to 3.3, 4.2 to 4.4, 5.2 to 5.4, 6.1 to 6.3.

#### 2. Statistics - I (45 hours)

**Preamble:** This course enables the student to understand the basic concepts and terminology in statistics. This course does not require any prerequisite. After completing this course, the students will be able to understand how frequency distribution are used in statistical analysis and identify the proper measure of central tendency to use for each level of measurement.

**Unit I:** Central tendencies - arithmetic mean - partition values - mode - geometric mean and harmonic mean - measures of dispersion (9 hours)

Unit II: Moments, Skewness and Kurtosis - Curve fitting - Method of least squares - Fitting lines- Parabolic, Exponential and logarithmic curves.(9 hours)

**Unit III:** Correlation and regression - Scatter diagram - Karl Pearson's coefficient of correlation - Properties - Lines of regression, Regression coefficient and properties - Rank correlation.

(9 hours)

**Unit IV:** Discrete Probability Distributions: Geometric, Binomial and Poisson distributions - Their moment generating function, Characteristic function, Properties and simple application.

(9 hours)

Unit V: Continuous Probability Distributions: Beta and Gamma Distributions, Normal distribution
Standard normal distribution - Their properties - Simple Problems - Importance of normal distribution.
(9 hours)

**Text Book:** S. Arumugam and A. Thangapandi Isaac, *Statistics* - New gamma publishing house, June 2007 (Chapters 2 (2.1 - 2.4) Chapter 3 (3.1), 4, 5,6,8 and 13)

## 3. Statistics – II (45 hours)

**Preamble:** This course focuses on the design and analysis of survey samples for finite populations. This course requires the knowledge of basic statistics as prerequisite. Upon successful completion of this course, the students will be able to test or analyze their sample data with the help of several tests or ANOVA tables they study in this course.

**Unit I:** Tests of Significance (Large samples): Sampling distribution - Testing of Hypothesis - Type I and Type II errors - Critical region, level of significance - Test of significance for large samples - Testing a single proportion - Difference of proportions - testing a single mean - Difference of means. **(9 hours)** 

Unit II: Tests of Significance (Large samples): Tests based on t - distribution - Single mean - Difference of means - Tests based on F - distribution.(9 hours)

**Unit III:** Test based on chi square distribution - Goodness of fit - Independence of attributes. **(9 hours)** 

**Unit IV:** Analysis of time series: Time series - Components of a time series - measurement of trends. **(9 hours)** 

Unit V: Analysis of Variance - One criterion of classification - Two criteria of classification - Three criteria of classification.(9 hours)

**Text Book:** S. Arumugam and A. Thangapandi Isaac, *Statistics* - New gamma publishing House, June 2007 (For Unit I - III - Chapters 14, 15 and 16, For Unit IV - Chapter 10, For Unit V - Chapter 17).

## 4. Numerical Methods (45 hours)

**Preamble:** The objective of this course is to develop the skills in solving algebraic, transcendental, differential and integral equations numerically. prerequisite. The prerequisite of this course is a basic knowledge of solving equations, differentiation and integration. The outcome of the course is enabling the students to get numerical (approximate) solutions wherever analytic (exact) solutions are not possible.

**Unit I:** Errors in Numerical Calculations: Errors and their computations - A general error formula - Error in a series. Approximation Solution of Algebraic and Transcendental equations: The Bisection method - The Method of False position - Iteration method - Newton - Raphson method. **(9 hours)** 

**Unit II:** Interpolation: Finite differences - Forward Differences - Backward Differences - Central Differences - Symbolic Relations and Separation of Symbols. Newton's Formulae for Interpolation - Gauss's central difference formulae - Stirling's formula - Interpolation with unevenly spaced points: Lagrange's interpolation formula - Inverse Interpolation.

(9 hours)

**Unit III:** Numerical Differentiation: Derivatives using Newton's Forward Difference Formula - Derivatives using Newton's Backward Difference Formula - Derivatives using Stirling's Formula - Maxima and Minima of Tabulated Function. **(9 hours)** 

**Unit IV:** Numerical Integration: General Quadrature Formula - Trapezoidal Rule - Simpson's 1/3 Rule - Simpson's 3/8 Rule. **(9 hours)** 

Unit V: Numerical Solutions of System of Linear Equations: Gauss elimination method - Gauss - Jordan method - Jacobi's method - Gauss - Seidel method.(9 hours)

#### **Text Book:**

S. S. Sastry, *Introductory Methods of Numerical Analysis*, Prentice Hall of India, Pvt. Ltd., New Delhi (Fourth Edition, 2005).

## 5. Linear Programming (45 hours)

**Preamble:** The objective of this course is to introduce the fundamental concepts of Linear Programming Problems (LPP) and to train the students in solving managerial and other real time problems. On the successful completion of the course, students will be able to know the origin and development of LPP, different techniques to solve it and the application of Transportation and Assignment problems.

Unit I: Definition - examples - Mathematical formulation - standard form - Theorems (statements only) - Graphical solution - simplex method.(9 hours)

**Unit II:** The Big-M method - Two phase simplex method

(9 hours)

**Unit III:** Duality - The dual of the dual is the primal - Duality theorems (Statements only) - Dual simplex method. **(8 hours)** 

**Unit IV:** Transportation problem - Mathematical formulation - North west corner rule - method of matrix minima - Vogel's Approximation method - MODI optimality test - Assignment problem.

(10 hours)

Unit V: Integer Programming: Gomory's cutting plane method - Branch and bound method.

(9 hours)

**Text Book:** Kanti Swarup , P.K. Gupta and Man Mohan, *Operations Research*, (2009) (Relevant Sections from Chapters 2, 3, 4, 6, 7 and 14).

## 6. Combinatorial Mathematics (45 hours)

**Preamble:** To introduce combinatorial techniques for solving enumeration problems. This course does not require any prerequisite except the basic ideas of nCr and nPr. On the successful completion of the course, students will be able to understand the important concepts of contemporary Combinatorics, find the way to count the number of ways in more than one way, and solve enumeration problems using combinatorial techniques.

**Unit I**: Selections and binomial coefficients - permutations - ordered selections - unordered selections. **(9 hours)** 

**Unit II**: Pairing problems - pairings within a set - pairing between sets - an optional assignment problem. **(9 hours)** 

**Unit III**: Recurrence - Fibonacci type relations - using generating functions.

(9 hours)

**Unit IV**: The Inclusion - Exclusion principle - the principle - Rook polynomials.

(9 hours)

Unit V: Block designs - square block designs.

(9 hours)

**Text Books**: Ian Anderson,. A first course in combinatorial mathematics.

#### 7. Coding Theory (45 hours)

**Preamble:** Coding theory is the study of methods for efficient and accurate transfer of information from one place to another. Pre-requisite for this course is elementary knowledge of linear algebra. On successful completion of this course, students will be able to apply the mathematical ideas of binary codes to engineering and computer science.

**Unit I:** Basic assumptions - correcting and detecting error patterns-information rate - effects of error correction and detection - finding the most likely code word transmitted.

(9 hours)

**Unit II:** Linear codes - two important sub-spaces-independence - basic, dimension - matrices - Bases for C and C<sup>+</sup> generating matrices on coding. **(9 hours)** 

Unit III: Parity check matrices - equivalent codes-distance of a linear code - Linear codes - cosets - MLD for linear codes - Reliability of IMLD for linear codes.(9 hours)

**Unit IV:** Some bounds for codes - perfect codes - hamming codes - extended codes - The extended Golay code - decoding the extended Golay code - Golay code. **(9 hours)** 

**Unit V:** Polynomials and words - introduction to cyclic codes - Polynomial encoding and decoding - finding cyclic codes - Dual cyclic codes. **(9 hours)** 

**Text Book:** Hoffman et.al., Coding Theory, the essentials, Marcel Dekker, Newyork.

(Chapters 1 to 4 except sections 3.8,3.9)

## 8. Operations Research (45 hours)

**Preamble:** The objective of this course is to provide students with basic skills and knowledge of operations research and to introduce students to practical application of operations research in big mining projects. On the successful completion of the course, students will be able to solve problems in sequencing, game theory, inventory and queuing theory and also understand and analyze networks using PERT and CPM.

**Unit I:** Inventory - deterministic models - uniform rate of demand, infinite state of production and no shortage - Uniform rate of demand, finite rate of replenishment and no shortage - Uniform rate of demand, instantaneous production with shortages. **(9 hours)** 

**Unit II**: Queuing Theory - General concepts and definitions - classification of Queues - Poisson process - properties of Poisson process models:

i. (M/M/1): (∞/FCFS)
 ii. (M/M/1): (N/FCFS)
 iii. (M/M/S): (∞/FCFS)

(9 hours)

**Unit III:** Network Analysis - Drawing Network diagram - critical path method - labelling method - concept of slack and floats on network - PERT - Difference in PERT and CPM.

(9 hours)

**Unit IV:** Non-linear Programming: General Non-linear Programming Problem - Problem of constrained maxima and minima - graphical solution - saddle point problems - saddle points and N. L.P.P. (9 hours)

Unit V: Non - linear Programming Techniques: Kuhn - Tucker conditions - Non-negative constraints - Quadratic Programming - Wolfe's modified simplex method - Beale's method - Separable convex programming.
 (9 hours)

**Text Book:** Kanti Swarup, P.K. Gupta and Man Mohan, *Operations Research*, (2009) Sultan Chand and Son Publications (Relevant Sections from Chapters 12, 13, 16, 17 and 19).

## 9. Mathematical Programming Using JAVA (45 hours)

**Unit I:** Introduction to JAVA - History - overview - JAVA application programs - JAVA Applets - commands line arguments - Data types - variables - comments. **(9 hours)** 

**Unit II:** Objects and Classes - Defining a class - constructors - multiple constructors - wrapper classes - conversion of data types - command live and keyboard input - Attributes and methods - Attributes - overriding - object composition with a simple example. **(9 hours)** 

**Unit III:** If structure - nested if structure - break and labelled break - switch structures while loop - do loop - for loop - natural sum and partial sum of series using loops - Divergent series verification using loops - nested loop. **(9 hours)** 

**Unit IV:** HTML entity encoding in JAVA - JAVA string encoding- attributes - encoding schemes that are supported by the JAVA platform - character encoding - objects , Images and applets in HTML documents. **(9 hours)** 

**Unit V:** JAVA script - Grammar - objects - functions - inheritance - arrays. (9 hours)

#### **Reference Books:**

- 1. C. Xavier, JAVA 2 programming, Scitech Publication
- 2. Herbert Schildt, The Complete Reference JAVA 2.

## 10. Integral transforms and applications (45 hours)

**Preamble:** The objective of the course is to introduce the basic concepts of Integral transforms and Fourier series. The pre-requisite is a sound knowledge of mathematics at the higher secondary level. On successful completion of the course, the learner will be able to solve problems by applying Laplace transform methods.

**Unit I:** The Laplace transforms: Definition - sufficient conditions for the existence of the Laplace transform - Laplace transform of periodic functions - some general theorems. **(9 hours)** 

Unit II: The inverse transforms. (9 hours)

**Unit III:** Applications to differential equations - solving simultaneous equations and differential equations with variable coefficients. **(9 hours)** 

Unit IV: Fourier series - even and odd functions - half - range Fourier series. (9 hours)

Unit V: Fourier Transform – Complex form of Fourier integral formula - Fourier integral theorem
 properties of Fourier transform – Fourier cosine and Fourier sine transforms and their properties.
 (9 hours)

**Text Book:** S. Narayanan and T. K. Manicavachagom Pillay, *Calculus - Volume III*, S. Viswanathan (Printers and Publishers ) Pvt. Ltd.2008 (Chapters 5 and 6 (Sections 1 to 4 & 9 - 12)

## 11. Special Functions (45 hours)

**Unit I:** The exponential function - the logarithmic function - definition of  $x^a$  – the trigonometric functions. (9 hours)

**Unit II:** Beta and gamma functions.

(9 hours)

**Unit III:** Applications of Gamma functions to multiple integrals.

(9 hours)

**Unit IV:** Legendre 's equation - solution - Legendre's function of the first and second kinds - Orthogonal properties of Legendre's Polynomial. **(9 hours)** 

**Unit V:** Bessel's equations and Bessel's functions - Definition and solution - Bessel's function of the first kind of order n - generating function- some trigonometric expansions involving Bessel's functions. **(9 hours)** 

#### Text Book:

- 1. R.R.Goldberg, *Methods of Real analysis*, Oxford and IBH Publishing, 1963 (For Unit I Chapter 8 Sections 8.2 to 8.4)
- 2. S. Narayanan and T. K. Manicavachagom Pillay, *Calculus* Volume II S. Viswanathan (Printers and Publishers) Pvt. Ltd. 2008 (For Unit II Chapter 7 Sections 1 to 5: For Unit III Chapter 7 Section 6).
- 3. U.P. Singh, R.Y. Denis, S. K.D. Dubey and K.N.Singh, *Differential Equations and Integral transforms*, (2005) Dominant Publishers and Distributors (For Unit IV 4.1 and for Unit V 4.2).

## 12. Mathematical Logic (45 hours)

**Unit I:** Mathematical Induction - Techniques of proof.

(9 hours)

**Unit II:** Mathematical logic - Statements and notations - connectives - statement formulas and truth tables - conditional and biconditional statements - well formed formulas. **(9 hours)** 

**Unit III:** Tautology - equivalence of formulas - duality law - principal disjunctive and conjunctive normal forms. **(9 hours)** 

**Unit IV:** Axiom of choice - Zorn's lemma - well - ordering - Zermelo's theorem - Schroder-Bernstein theorem. **(9 hours)** 

Unit V: Ordinals and Cardinals. (9 hours)

#### **Text Book:**

- 1. M.K. Venkataraman . N. Sridharan and N. Chandrasekaran, *Discrete Mathematics*, The National Publishing Company (Edition June 2006 ) (For units I, II and III).
- 2. Paul.R. Halmos, *Naïve set theory*, Springer International Edition (1960) (For Unit IV Chapters 15 to 17 and 22; For Unit V Chapters 19 to 21,24 and 25).

## 13. Astronomy (45 hours)

**Preamble:** Astronomy is the science that deals with the material universe beyond the earth's atmosphere. It studies the nature and constitution of heavenly bodies. This course focuses on the topic spherical trigonometry, the zones of earth, refraction laws of refraction, geocentric parallax and Kepler's laws. On the successful completion of the course, the student is able to know all celestial bodies and their movements in the galaxy.

Unit I: Spherical Trigonometry - Celestial sphere- Diurnal motion. (9 hours)

**Unit II**: Earth - the zones of Earth - Terrestrial latitudes and longitudes - radius of earth - rotation of earth - Dip of horizon - Twilight. **(9 hours)** 

**Unit III**: Refraction laws of refraction - effects of refraction - cassini's formula horizontal refraction. **(9 hours)** 

**Unit IV**: Geocetric parallax - effects - horizontal parallax of moon - angular diameter - comparison of geocentric parallax and refraction. **(9 hours)** 

**Unit V**: Kepler's laws - longitude of perigee - eccentricity of earth's orbit - verification of Kepler's laws in the case of Earth Newton's deductions form Kepler's laws mean anomaly -Geocentric and heliocentric latitudes and longitudes. **(9 hours)** 

**Text Book:** S. Kumaravelu and Susheela Kumaravelu, *Astronomy* 

## 14. MATHEMATICAL MODELING (45 hours)

**Unit I:** Linear growth and Decay Models - Non linear growth and Decay Models - Compartment Models - Dynamics Problems - Geometrical Problems. **(9 hours)** 

**Unit II:** Population Dynamics - Epidemics - Compartment Models - Economics, Medicine, Arms Race, Battles and International Trade. **(9 hours)** 

**Unit III**: Planetary Motion - Circular Motion - Motion of Satellites - Modeling through Linear difference equations of Second Order. **(9 hours)** 

**Unit IV**: Basic theory of difference equation with Constant Coefficients - Economics and Finance - Population Dynamics and Genetics - Probability Theory. **(9 hours)** 

**Unit V**: Solutions that can be Modelled through graphs - Models in terms of directed graphs, signed graphs, weighted digraphs and unoriented graphs. **(9 hours)** 

#### **Books for Reference:**

1. Kapur. J.N. - Treatment as in "Mathematical Modeling" by J.N.Kapur - New Age International Publishers, 2004.

## Semester VII to X

Syllabus and scheme of examination as per that of the 2-year M.Sc (Mathematics) Degree program of the University Department of Mathematics which is in force at that time.

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